Short-run and long-run equilibrium

Consider a market that is in a long-run equilibrium. In this equilibrium, each firm's short-run and long-run total cost functions are given by:

$$SRTC(q) = q^3 - 3q^2 + 3q + 4$$
$$LRTC(q) = 3q$$

The market demand is given by $Q_D(P) = 27 - P$.

1. What is the equilibrium price in the initial long-run equilibrium?
2. Calculate the quantity each firm produces [Hint: If the market is in a long-run equilibrium, it is also in a short-run equilibrium.]
3. What is the equilibrium market quantity in the initial long-run equilibrium? How many firms are in the market?
4. Derive each firm's supply curve (expressing it as a function of p). Derive the short-run market supply curve.
5. The items suddenly come into vogue, and the market demand shifts to $Q'(P) = 57 - 3P$. What are the equilibrium price and quantity in the short run?
6. If the market demand stays at Q' thereafter, how will the market adjust? How many firms will there be in the long run?

Solutions

1. Since the market is competitive, the equilibrium price must equal the minimum long-run average cost. $\frac{LRTC}{q} = 3$ this means $P^* = 3$

2. The short-run marginal cost has to equal the market price we found above. This gives $SRMC(q^*) = P^* \Rightarrow 3q^{*2} - 6q^* + 3 = 3 \Rightarrow 3q^*(q^* - 2) = 0$ so we find that $q^* = 2$ or $q^* = 0$ but $q^* = 0$ is a minimum.

3. Plugging the equilibrium price into the demand curve, we get $Q^* = 24$. Furthermore, since every firm produces 2 units, this means: $n * q^* = Q^* \Rightarrow n = 24/2 = 12$

4. The short-run supply curve is the marginal cost curve ($P = MC(q)$), in the range where $P \geq AVC(q)$; otherwise, the firm prefers to shut down. Solving $P = MC(q)$ for q gives:

$$P = 3q^2 - 6q + 3$$
$$\Rightarrow q = 1 + \sqrt{\frac{P}{3}}$$

However, we need that $P = MC(q) \geq AVC(q)$. We calculate average variable cost$AVC(q) = q^2 - 6q + 3$, Then $3q^2 - 6q + 3 \geq q^2 - 3q + 3$, which implies: $q \geq 3/2$, so $1 + \sqrt{\frac{P}{3}} \geq 3/2$, finally $P \geq 3/4$.

$$q(p) = \begin{cases} 1 + \sqrt{\frac{P}{3}} & \text{if } p \geq 3/4 \\ 0 & \text{if } p < 3/4 \end{cases}$$

This means the short-run market supply is:

$$Q(p) = \begin{cases} 12 + 12\sqrt{\frac{P}{3}} & \text{if } p \geq 3/4 \\ 0 & \text{if } p < 3/4 \end{cases}$$

5. We need to find the intersection between the market supply curve that we found before and the new demand curve: $12 + 4\sqrt{3P} = 57 - 3P$ which simplifies to $P = \frac{25}{3}$. This implies that $Q = 32$.

6. Because nothing has changed on the supply side, the new entering firms would drive the price back down to $P = 3$ and to find the quantity we just have to evaluate the new demand at the equilibrium price $P = 3$, so we obtain a quantity of $Q = 48$. Since each firm produces 2 units at this price, there have to be 24 firms.